Anritsu

**Application Note**

# Three and Four Port S-parameter Measurements

Scorpion®

*Calibrations and Mixed-Mode Parameters*

# Introduction

*Calibrations are the critical first step to multiport vector network analyzer (VNA) measurements and there are many choices. A mixed-mode S-parameter representation for balanced devices is also increasingly important and relatively easy to understand.*

With the increasing use of multi-port and balanced devices, the need for quality three and four port calibrations has increased in both coaxial and fixtured environments. Also the need for the presentation of S-parameter data in a mixed-mode format for differential circuitry has become apparent. This note will describe the multiport calibration algorithms available in certain versions of the MS462xx and the mixed-mode S-parameters that can be displayed using those calibrations. Some of the discussions will also apply to two port instruments but the document is tailored more toward the 3 port versions of the MS462xB and the 4 port MS462xD. The focus on the calibrations will be on ensuring minimum uncertainty with a reasonable amount of effort and the discussion on mixed mode will center on the definitions, data interpretation and examples.

## Calibrations

The first step in performing much of any three or four port analysis is in the calibration. Mixed mode S-parameters, impedance transformation [1], embedding and de-embedding [2] tend to have little value if the measurements upon which they are based have not been performed under a good calibration. It is assumed that the user is familiar with common two port calibration techniques (or at least the ones they use) although parts will be reviewed now. Recall that the controlling concept in S-parameter calibrations is to determine the artifacts of the measurement system by measuring a number of known standards (such as thru lines, transmission lines, loads, shorts and opens) (e.g., [3]-[7]). The various algorithms mainly differ in what standards they use and each combination has its advantages under certain conditions.

Starting with a two port VNA, the model has long been that of error boxes into which all of the instrument's non-idealities have been lumped [3] (see Figure 1). If the parameters corresponding to these boxes can be found during calibration, it is then straightforward to remove their effects from measured data to present the corrected result. This process can be thought of as de-embedding in some sense and mathematically involves simply solving a system of linear equations (this linearity assumption will be important).

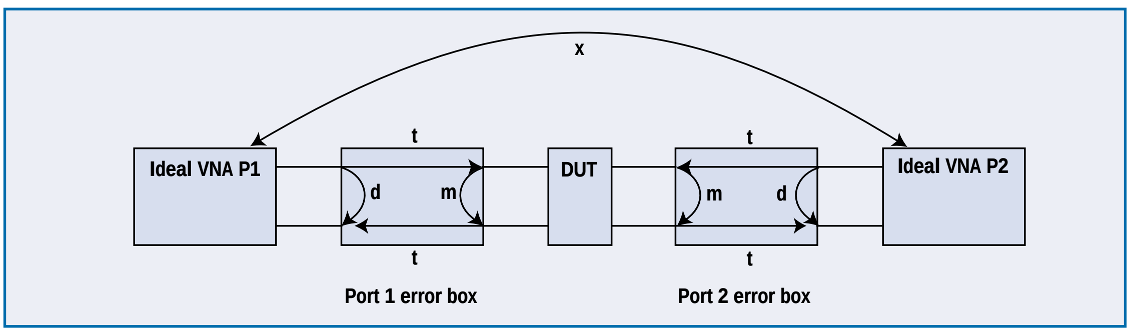


***Figure 1.*** *A simple and often used error model for a two port VNA is shown here. The non-idealities of the VNA are lumped basically into 2 error boxes with defined parameters. The lines on each port are used to delineate incident (top) and reflected (bottom) waves, they are not physical. The small letters denote the type of defect being corrected: t- tracking (some amplitude and phase error just on transmission or reflection, usually related to test set and converter loss), m- match, d- directivity, and x- isolation.*

The purpose of the calibration is to quantify each of these error terms through measurement of standards. As an example, a thru line connected between the ports will give information to help determine transmission tracking and connection of a perfect load to a port will give information on directivity (because if the load is perfect, there are no reflections so any measured reflection must be a directivity leakage). Isolation will be discussed in passing in the following sections although it is frequently ignored in practice. This is for two reasons (1) the internal leakages in the VNA are very small and (2) isolation due to packaging or the environment is very difficult to correct for. The latter follows since the leakage near the DUT is generally highly dependent on impedances present and on details in the environment (e.g., where exactly are the probe heads in a wafer prober) so the leakage signal will likely change between calibration (when loads are attached) and measurement (when the DUT is attached). In such a situation, attempts at isolation correction will usually makes things worse. Some details regarding how the other terms are computed are discussed in the appendix and in the references. There are many choices on exactly what standards are used and this has lead to a proliferation of calibration algorithms. Some of the more common ones are listed below along with their attributes.

## SOLT (a default selection in coaxial environments)

Uses a short, open and load on each port together with a thru line between ports 1 and 2. The load may be fixed or sliding (used to improve the ultimate residual directivity [6]). This is the most common calibration selection and usually works well for coaxial systems. The algorithm uses models for the standards (particularly the open) requiring characterization of those standards (usually by the manufacturer). The sequence of connection events in a typical SOLT cal is shown in Figure 2.

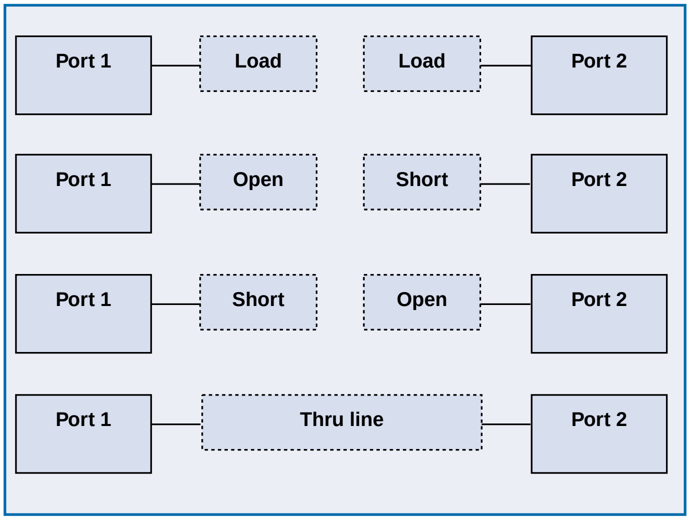


*Figure 2.* *The sequence of connection events for an SOLT cal is shown here. Usually the reflection connections are done two ports-at-a-time in order to save time although they can be done individually.*

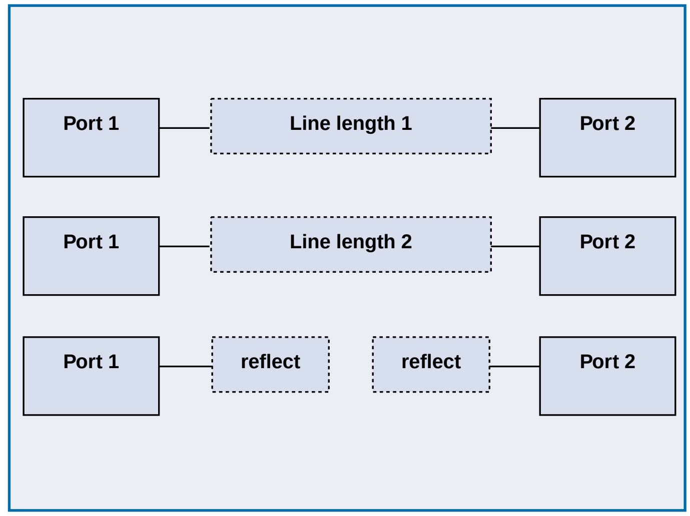


*Figure 3.* *The sequence of connection events for LRL is shown here. The reflect standard can be almost anything although shorts or opens are normally employed. Two lines are used when a limited bandwidth (~9:1 frequency ratio is the maximum) is required; multiple lines can be used for a larger bandwidth.*

## LRL

Uses a reflect standard (often a short) at each port together with two different line lengths connected between ports 1 and 2. The line lengths are chosen so that their difference is electrically significant but less than a half-wavelength at the highest frequency of interest. This method is particularly valuable for on-wafer and other testing where open standards are difficult to implement. It is somewhat band-limited and may require additional line standards for wider bandwidths [3]-[4]. In some sense, it is the cleanest cal since it only requires theoretically ideal transmission lines and an unknown reflection [7]. This method makes some use of redundancy in measurements and hence requires fewer standards connections than does SOLT and some other methods. The sequence of events when only 2 lines are needed is shown in Figure 3.

## LRM

Uses a reflect standard and a fixed load at each port together with a line between ports 1 and 2. Similar to LRL in its value for on-wafer test, but simpler to implement and much easier to use at lower frequencies and in coaxial environments. The connection events are shown in Figure 4.

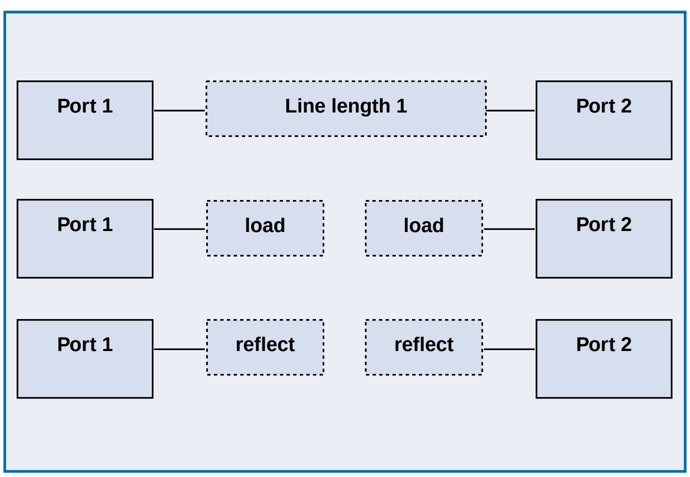


*Figure 4.* *The sequence of connection events for LRM is shown here. The use of the load gives much greater bandwidth but it must be modeled correctly at higher frequencies (mm-wave range primarily).*

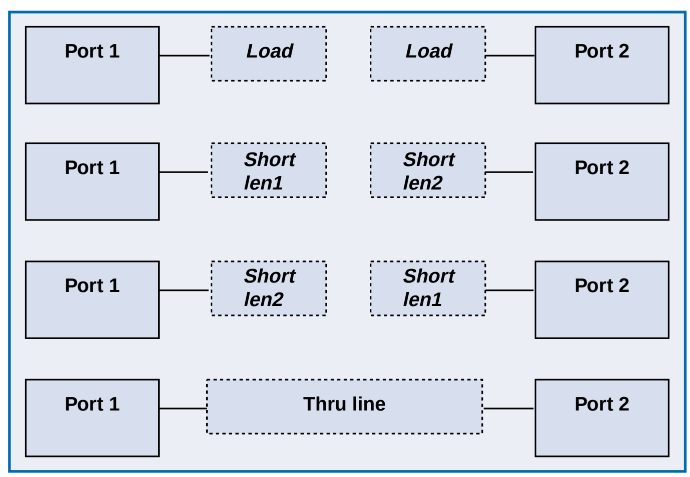


*Figure 5.* *The sequence of events for an offset short cal is shown here. The two different lengths of shorted line are used in lieu of an open and short for SOLT. As with LRL, this imposes a bandwidth limitation that can be overcome with additional standards.*

## Offset Short

Uses a load and two shorts (different offset lengths) at each port together with a thru line between ports 1 and 2. This method is often used for waveguide systems (since opens and high quality thrus are not required) and is bandlimited. The bandwidth can be extended using additional offset shorts (of different lengths obviously). The sequence of events is shown in Figure 5.

## AutoCal®

An automated calibration system that requires just connecting the autocal box to ports 1 and 2 (as well as connecting power and serial communications cables). The box contains internal impedance and thru standards that enable a transfer calibration in 1 user step. The primary advantage is improved time efficiency in a higher measurement throughput environment. It is actually a transfer standard between some original calibration (used when the module was characterized) and the instrument calibration [8].

Whatever method is chosen, the outcome of the cal is the determination (vs. frequency) of the terms shown in Figure 1. When the cal is applied, these error boxes are mathematically removed (de-embedded) from the raw measured data to generate corrected data.

## Multiports

The next question then is how to handle a system with more than 2 ports. Since an S-parameter ($S_{ij}$) would appear to involve only two ports (i and j), the obvious answer would be to perform a two port cal (as discussed above) between ports i and j and make the measurement. The other N-2 ports could be ignored and the measurement protocol, although lengthy, would be straightforward.

If one looks more closely at the definition of $S_{ij}$, however, it states that port j is driving and ALL OTHER PORTS are terminated in the reference impedance. Since all of the ports of the measurement system excluding i and j are probably not perfectly matched, these represent additional error terms that can, and often should, be corrected (see Figure 6). These terms are called load match terms and are more important for devices that are multilateral (significant transmission between more than 2 ports: couplers, dividers,...) than for devices that are more single-path in nature (diplexers, triplexers,...).

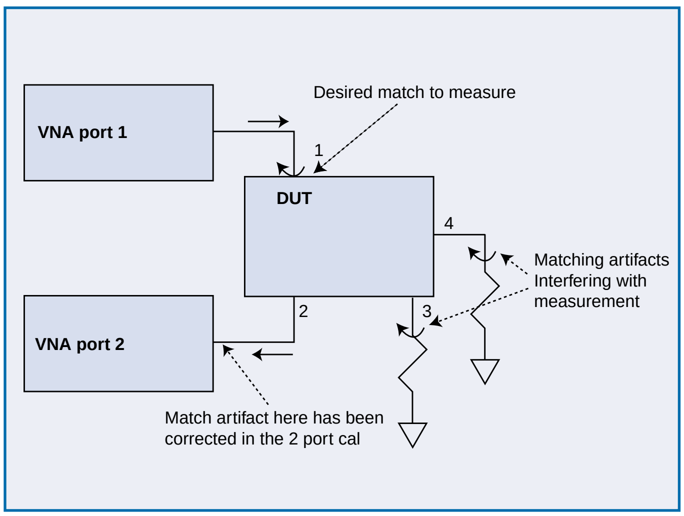


***Figure 6.*** *Illustration of an $S_{21}$ measurement on a 4 port device, using only a two port cal, is shown here. The curved arrows represent uncorrected reflections; the one at port 1 of the DUT represents the inherent reflections off the DUT. While reflections off VNA port 2 are corrected by the cal (since it is not a perfect match either), those off the terminations at ports 3 and 4 (which may be other VNA ports) are not corrected. If the transmission from port 1 to port 3 or 4 within the DUT is substantial, then ignoring the load matches can result in substantial error on $S_{21}$ (or $S_{11}$ for that matter).*

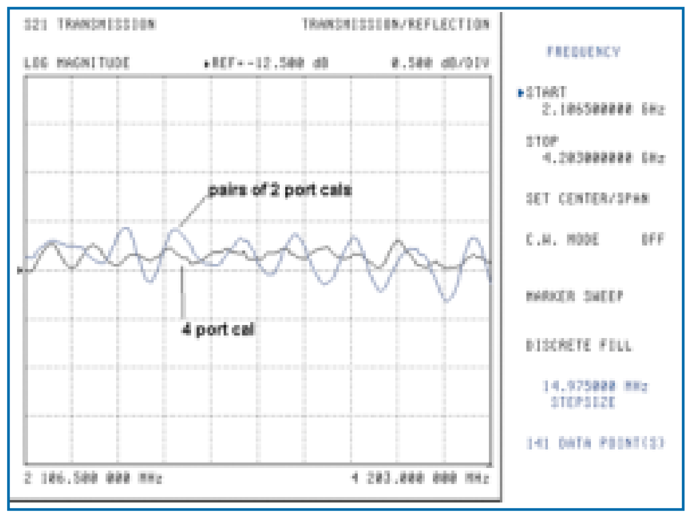


***Figure 7.*** *Example four port measurement of $S_{21}$ using a full 4 port cal or just using a two port cal. Since the transmission levels $|S_{31}|$ and $|S_{41}|$ are about the same as $|S_{21}|$, the error in ignoring the load matches can be grave. The residual ripple on the 4 port measurement reflects actual DUT behavior.*

As an example, the measurement of $|S_{21}|$ of a 4 port hybrid is shown in Figure 7 using both a full four port cal and using a two port cal (between ports 1 and 2). The unused ports in this case were terminated in raw matches of about –20 dB. The resulting error in using the 2 port cal and ignoring these load matches approaches 0.3–0.4 dB. This device is quite multilateral and will tend to show more significant problems.

In contrast, the results for a less multilateral diplexer are shown in Figure 8. Since the common node is the only one capable of seeing both other nodes, it might be expected that reflection measurements at this common node might have the most trouble and indeed that is clear. If this measurement is not a criticality, then a pair of two ports approach may suffice for such a device. In this plot a cal between ports 1 and 2 was again used and port 3 was terminated in about a –20 dB match.

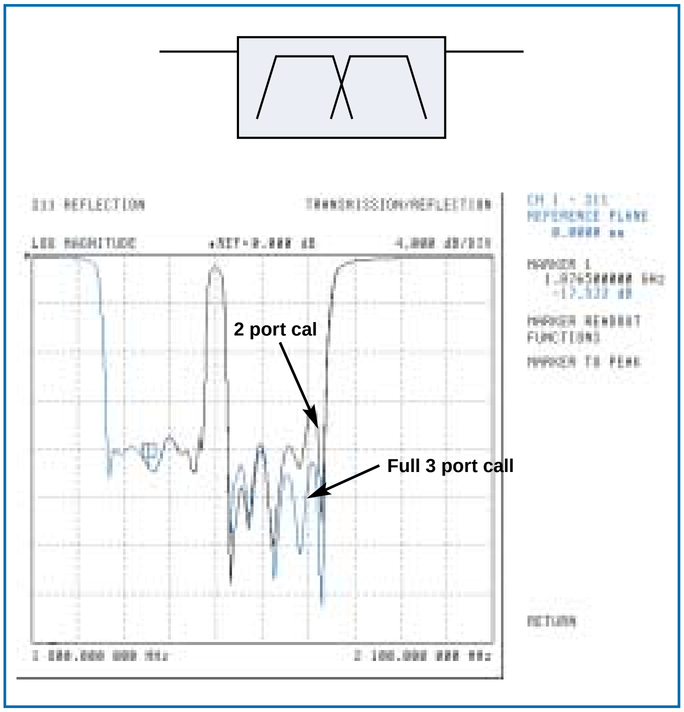


*Figure 8.* An example 3 port measurement using a full 3 port cal (lower trace) or a two port cal (upper trace) is shown here. Since this is a diplexer, $|S_{31}|$ is low when $|S_{21}|$ is high and vice–verse so the transmission coefficients are not severely affected by ignoring load match (as long as it is not extremely poor). Common node match, however, is still sensitive since it will directly measure raw load match in the passband of the uncorrected port.

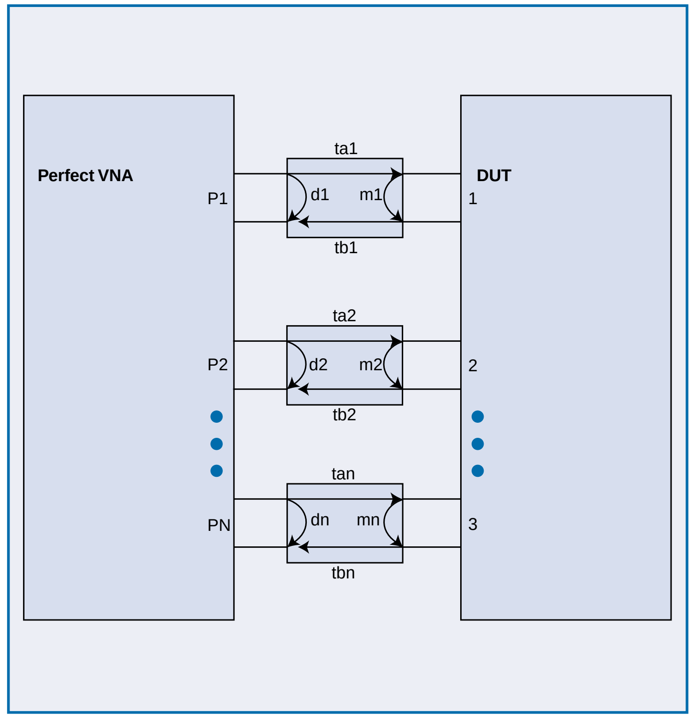


*Figure 9.* An N port error model is shown here. Directivity, match and reflection tracking can be assigned on a per–port basis while transmission tracking can be assigned an a pair-of-ports basis. Isolation is also nominally assigned on a pair-of-ports basis but that does have problems (isolation is often ignored practically).

We have explained why a two port cal can sometimes not suffice for an N port device but have yet to explain exactly what a full N–port cal is. Conceptually, it is really a straightforward extension of Figure 1 (e.g., [5]). The directivity and match terms discussed earlier are basically functions of the port itself so one must just add these terms for the additional ports. Tracking is usually broken down into reflection tracking (the offset incurred on a reflection measurement) and transmission tracking (the offset on a transmission measurement). Reflection tracking then also is a port property and can be assigned for each additional port. Transmission tracking is a property of a pair of ports (the correction to get a thru line to measure $S_{ij}=1$ after other port errors have been handled) hence this term must be assigned for every permutation of two ports. Finally isolation is usually also considered the property of a pair of ports so the same permutation will be required. The last statement is not entirely valid if isolation is dependent on DUT impedances and is normally reserved to correct for isolation deep in the instrument. This distinction will be ignored for now since the isolation step is normally skipped with modern equipment. Thus we have a fairly straightforward new error model as shown in Figure 9. As before, the pair of lines per port are to help delineate incident and reflected waves. The reflection tracking for port *q* can be thought of as the product of *taq* and *tbq*; the transmission tracking from port *k* to port *l* can be thought of as the product of *tak* and *tbl* (no new information being added, just redistributing it).

When going to 3 and 4 port cals from a two-port cal, the algorithm choices are theoretically as varied as before. We will cover only two here: SOLT and TRX. SOLT is identical to its two port cousin in that it needs three reflection standards per port plus thru line connects to at least one other port (port 1 normally; the extra ports do not necessarily have to be connected to the same base port but that is usually done as well). Additional thru connects are allowed if it is desired to avoid the use redundancy (this is discussed later in this section). TRX is a multiport extension of LRL modified for practical VNA architectures. It requires only one reflection standard per port (a short normally) and at least one thru line per additional port (again to port 1 normally). TRX uses the information from the thru connect to compute reflection tracking and directivity of the additional port (where SOLT uses the additional reflection standards to get these). The short is still required to extract source match although, in principle, other reflection standards could be used.

The algorithms are setup so that one performs a full two port cal first (using any of the algorithms described earlier) and then completes the steps for a three or four port cal:
*3 port cal*
*SOLT: thru (1-3), open on 3, short on 3, load on 3*
*TRX: thru (1-3), short on 3*

*4 port cal*
*SOLT: thru (1-3), thru (1-4), open on 3-short on 4, short on 3-open on 4, loads on 3 and 4*
*TRX: thru (1-3), thru (1-4), short on 3, short on 4*

These steps are shown pictorially in Figure 10 for SOLT.

It is also possible to complete a three port cal before going to a four port cal in which case the number of connections required on the four port step will obviously be reduced. Also it is possible to perform just a three port cal on a four port instrument if that is all that is needed.

The isolation port of the cal is handled depending on how the base 2 port cal was performed. If isolation was included on the 2 port cal, it will be done on the N-port cal. If it was skipped during the 2 port cal, it will not be done on the N-port section. For the isolation step at N-port, isolation devices (normally loads) would be connected to all N ports since every transmission path must be measured.

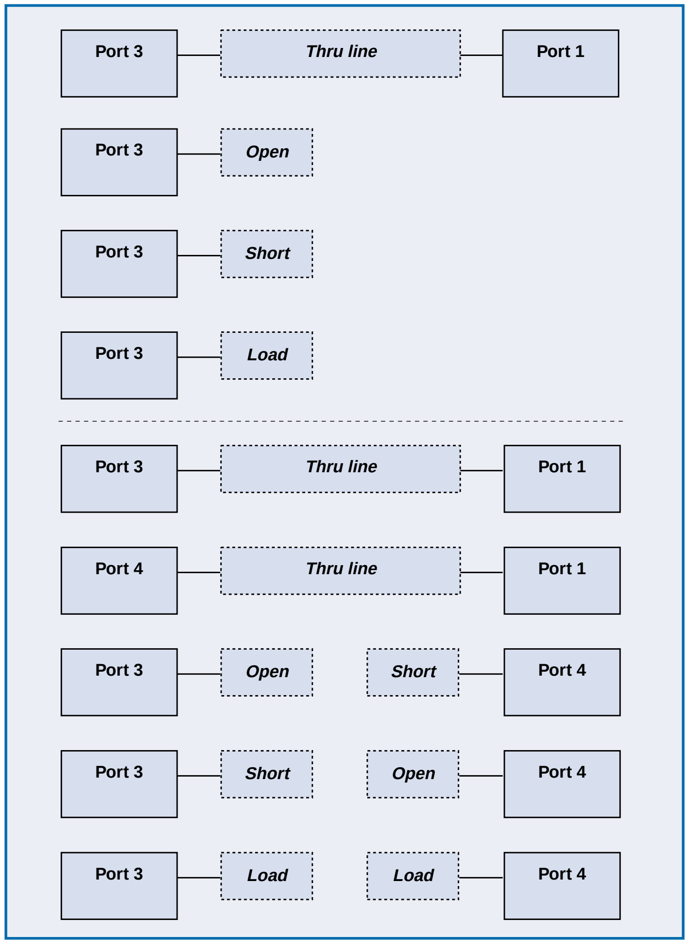


***Figure 10.*** *The finishing steps for a three port (top) and a four port (bottom) cal are shown here. An SOLT cal type was chosen for this example. Isolation and extra thru line connects have been omitted.*

Although touched on in reference to two port calibrations, this issue of redundancy becomes far more important as the number of ports increases. On initial inspection, it would appear that N(N-1)/2 thru line connections are needed to get all of the transmission tracking terms since there are that many permutations of N ports. This is not the case, however, since the tracking terms are not entirely independent. In general, a total of N-1 thru lines are the minimum needed and, if the cal is done carefully, it will be sufficient to get all of the terms (this issue is explored in more detail in the appendix). While this can save calibration steps, it does mean a lack of care taken in one step of the calibration can propagate to many terms and in, some cases, multiply. Thus the use of redundancy can be viewed as being traded off against care required during the calibration. The amount of care required, in turn, is dependent on the necessary cal performance required. Asking for the measurement of a –20 dB match with < 0.2 dB uncertainty will require far greater care than the measurement of a –10 dB match with < 0.1 dB uncertainty. Care in this case refers to monitoring connector quality and repeatability, torque consistency, cable stability, temperature stability, and cal component quality (accuracy and age of characterization) among other factors.

| Number of Ports | Required Thru-line Connects | Optional Thru-line Connects |
|---|---|---|
| 2 | 1-2 | |
| 3 | 1-2, 1-3 | 2-3 |
| 4 | 1-2, 1-3, 1-4 | 2-3, 2-4, 3-4 |

### Miscellaneous calibration issues: connectors and reference planes

Much of the standard advice on the use of high quality, phase stable cables and good quality connectors applies. Another common area of concern is that of reference planes: the positions (relative to each port) where 0 phase is defined. Because of the nature of the mixed mode computations discussed in later sections, it is particularly important to correctly establish reference planes for these measurements.

- Reference planes are normally established during the reflection measurements. If more than one (e.g., for SOLT), they must be done at the same plane.
- Any length offsets for the cal standards must be entered correctly
- If a non-zero length thru is used, its length must be entered correctly. This point frequently causes confusion. In a two port cal, if the reflection measurements for port 1 are done at a male interface and those for port 2 are done at a female interface, then the two interfaces can simply be connected to together for the thru and this will be zero length. If, for example, the reflection measurements are done at the same type of interface and some adapter is needed for the thru, then this is not a zero length thru and the length must be entered. If a phase equal insertable is used for this process (see Figure 11), its length is precisely known.

To keep zero length thrus when the final interfaces must all be of the same type (e.g., the DUT ports are all SMA female), one can use phase equal insertables in a different way as shown in Figure 12.

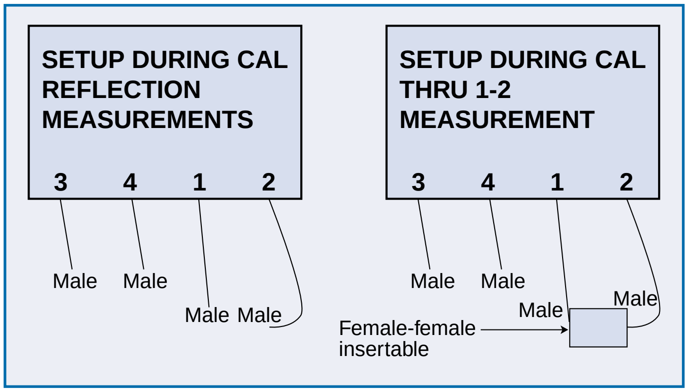


*Figure 11.* *The use of a non-zero length thru in a calibration is shown here. If all reference planes need to be the same type, one option is to use an insertable (or other adapter) to form the thru. The adapter's electrical length must be known and entered as the thru length during the cal. Phase equal insertables (available in many cal kits) have precisely known lengths.*

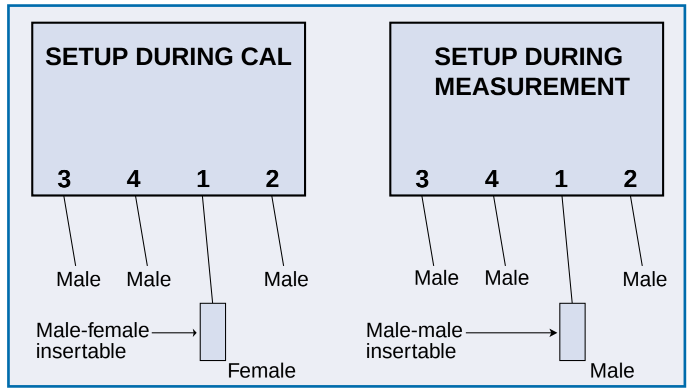


*Figure 12.* *One use of phase-equal insertables is shown here. The female end of the port 1 cable is maintained during the cal so zero length thrus can be maintained (just connect the cables). This port can then become a male connection during the measurement of the all-female connectorized DUT. Since the insertables have precisely the same phase length, the reference planes are maintained in the same location.*

## Calibrations and fixturing

In the N-port realm, it is sometimes desirable to perform calibrations at fixture level. While there are major traceability issues in doing so, the benefits (in terms of measurement quality) can be substantial if the fixture is electrically complex. To understand this, consider the calibration at the coaxial (outside) plane and what must be done to the data to extract the true performance of the device. If the fixture can be reasonably modeled as a lossless transmission line of 50 ohm impedance (or whatever the calibration impedance may be, see the impedance transformation app note for more information[1]), the correction is just a simple reference plane extension and there should be few complications. If the loss of the fixture cannot be neglected but is flat with frequency, the correction is still quite simple.

Beyond these two cases, the problem evolves into more of a full de-embedding issue in which case the S-parameters of the fixture must be well-modeled or measured (which is covered in a separate application note [2]). The modeling may occur via multiple simulation passes with comparison to measurement or with other techniques but it can be complex. There are situations where that approach is practical and others where it is more desirable to perform the calibration at or near the DUT reference planes. The latter is more attractive if the DUT planes are stable and accessible (e.g., in a wafer probe environment or a highly repeatable fixture) and there are not mechanical problems with generating cal standards for the environment near the DUT plane. Another issue may be that of required traceability which becomes more difficult for self-generated cal standards.

This raises the issue of how the cal standards should be generated which is, in turn, dependent on the cal algorithm chosen. The generation of a short raises few problems in the RF frequency range; the link to ground must simply be robust enough that the inductance is low. The reference plane must, of course, be considered and must be consistent among all standards. An open (required for SOLT) is more complicated in that its capacitance can almost never be neglected and hence must be characterized. The very open environment of many fixtures makes this somewhat more difficult in that the fields radiated from an open may be large enough that the open capacitance will be affected by neighboring structures (including possibly the user). At any rate, the open can be characterized sufficiently with a frequency dependent polynomial (usually a cubic will suffice) and this can be determined through modeling or de-embedding. The difficulty with the open is one reason for migrating to LRL/LRM/TRX type cals for the fixtured environment. Thru lines, a reflection (often a short) and possibly a load will be required. The load can be traded for an extra thru line (LRM<->LRL) as discussed earlier and may be preferred if it is difficult to squeeze a longer line length into the fixtured environment. The only major complication with the load is there may be excess inductance although that is often not a problem at RF frequencies. If it is an issue, a simple model is just a series inductance-resistance combination (although the real part may need to have some frequency dependence as the frequencies increase). The thru line model typically assumes a perfect match so it is important that discontinuities be minimized on any line runs between reference planes. Some example structures are shown in Figure 13.

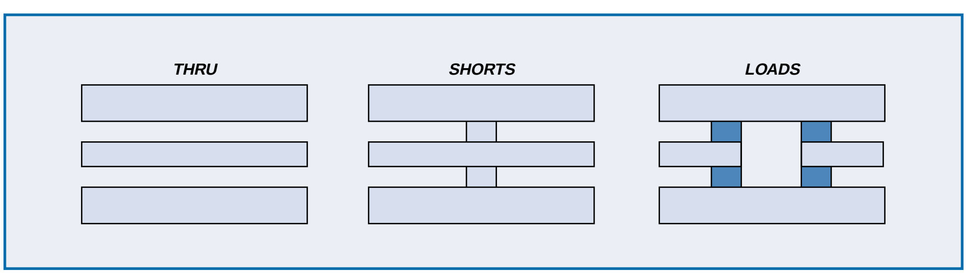


*Figure 13.* *An example set of standards for a wafer-based LRM cal are shown here. The short size is a bit exaggerated for clarity but the inductance to ground must not be too high for the frequencies involved. The load is generally constructed with parallel 100 ohm resistances to maintain the mode and reduce inductance.*

### Mixed Mode Parameters

The reader is no doubt familiar with the concept of a differential amplifier and the general concepts of differential and common-mode signals [9]. It is clear that it may be useful to represent S-parameter data in a form corresponding to differential drive/reception (and similarly for common-mode). Such a formulation has been developed in the past (e.g., [9]-[11]) and, more recently, instruments capable of displaying data in such a representation have become available. The purpose of this section is to describe and define this representation, provide some hints on data interpretation and deal briefly with uncertainty analysis in this slightly different parameter world.

Balanced devices are becoming increasingly common in modern wireless and other devices. The reasons for this trend are many-fold but include better noise immunity, more efficient use of power (i.e., longer battery life), lower cost (e.g., no mixer baluns), smaller size and harmonic rejection. Certain structures such as many mixers and A/D converters are naturally balanced thus making circuit design somewhat simpler if all devices in the chain are balanced. While this trend has begun in the IF sections of many radios, it is certainly propagating up the RF chain and the proper characterization of many different types of balanced devices is increasingly important. Example receiver structures for both single-ended and balanced topologies are shown in Figure 14.

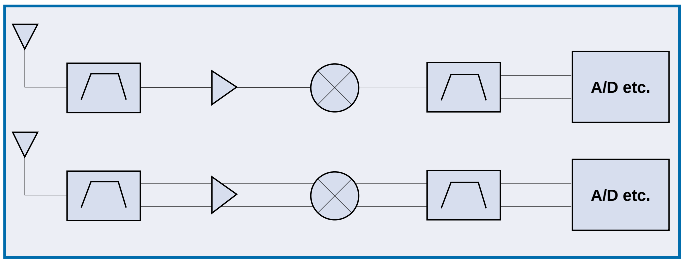


***Figure 14.*** *A single ended (top) and balanced receiver architectures are shown here. While any subset could be balanced, the IF sections are often the first to be converted.*

A very important assumption is that device operation is linear with respect to drive type. That is, the drive signals on the various ports can be linearly superimposed. This is critical to how the differential and common-mode parameters will be generated.

A logical way to measure many of these parameters would be to physically drive a device with differential and common mode signals and have the receivers setup correspondingly. This is, however, very difficult to do over large bandwidths with a sufficient degree of balance as is typically required of instrumentation. Also this would require the creation of very specialized calibration kits with concomitant traceability issues. An approach that is often used, instead, is to measure 4-port single ended S-parameters and perform a conversion. The conversion will generate results as if the ports were being driven in pairs.

Instead of each of the four ports being driven in turn, the key to the transformation is to think of the ports being driven two at a time. Each pair (either 1-2 or 3-4) can be driven either in phase (common mode drive) or 180 degrees out of phase (differential drive). For the transformation, it is convenient to group together single ended ports 1 and 2 together as the new port 1 (which can be driven differentially, common-mode, or some combination). Similarly, single ended ports 3 and 4 will be grouped as the new port 2 (same idea: differential, common-mode, or combination). Thus the new basis is to think of a port pair as being driven in phase or 180 degrees out of phase (instead of thinking of each port of the pair being driven individually). The new input and output bases are illustrated in Figure 15. The reader may recognize this as a simple shift in basis, which can be thought of as a 45 degree rotation.

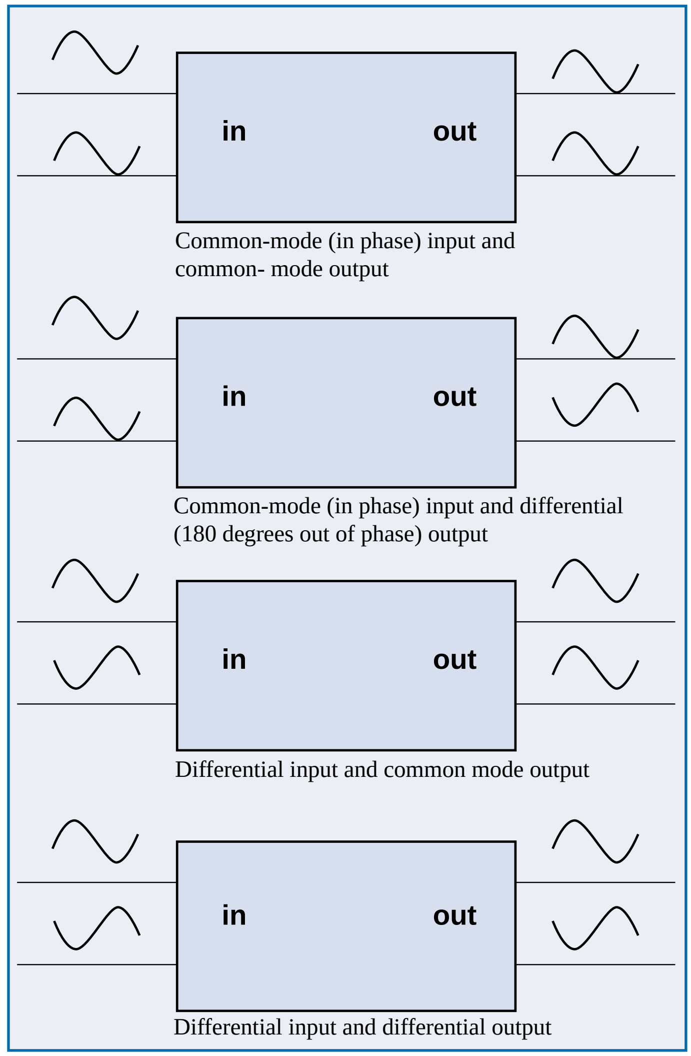


***Figure 15.*** *The new bases for analyzing mixed-mode S-parameters are shown here. With the physical ports considered as pairs, one can analyze in terms of common-mode and differential drive and common-mode and differential output.*

The next step is to construct S-parameters for this type of input/output. The possible incident waveforms can be deduced from Figure 15: differential on the new port 1, common-mode on port 1, differential on port 2, and common-mode on port 2. The output waves will have the some structure which leads to the S-parameter matrix structure depicted in Eq. 1.

$$\begin{bmatrix} b_{d1} \\ b_{d2} \\ b_{c1} \\ b_{c2} \end{bmatrix} = \begin{bmatrix} S_{d1d1} & S_{d1d2} & S_{d1c1} & S_{d1c2} \\ S_{d2d1} & S_{d2d2} & S_{d2c1} & S_{d2c2} \\ S_{c1d1} & S_{c1d2} & S_{c1c1} & S_{c1c2} \\ S_{c2d1} & S_{c2d2} & S_{c2c1} & S_{c2c2} \end{bmatrix} \begin{bmatrix} a_{d1} \\ a_{d2} \\ a_{c1} \\ a_{c2} \end{bmatrix} = \begin{bmatrix} S_{dd} & S_{dc} \\ S_{cd} & S_{cc} \end{bmatrix} \begin{bmatrix} a_{d1} \\ a_{d2} \\ a_{c1} \\ a_{c2} \end{bmatrix}$$

Eq. (1)

Here the four S blocks in the square matrix to the left are actually submatrices [9]. $S_{dd}$ corresponds to the four purely differential S-parameters while $S_{cc}$ corresponds to the four purely common-mode parameters. The other two quadrants cover the mode-conversion terms. These are expanded in the center part of Eq. 1. The matrix equation can be interpreted as expressing an output wave $b_i$ in terms of the four possible input waves $a_{d1}$, $a_{d2}$, $a_{c1}$, and $a_{c2}$. An example of one of these sub-equations is shown below.

$$b_{d1} = S_{d1d1}a_{d1} + S_{d1d2}a_{d2} + S_{d1c1}a_{c1} + S_{d1c2}a_{c2}$$

Eq. (2)

It is straightforward to write the relationships between the single-ended incident and reflected waves and the new balanced versions. The difference and sum choices are obvious for differential and common-mode waves, respectively, and all else that is needed is a normalization factor to keep power levels equivalent.

Eq. (3)

$$a_{d1} = \frac{1}{\sqrt{2}}(a_1 - a_2)$$

$$a_{c1} = \frac{1}{\sqrt{2}}(a_1 + a_2)$$

$$a_{d2} = \frac{1}{\sqrt{2}}(a_3 - a_4)$$

$$a_{c2} = \frac{1}{\sqrt{2}}(a_3 + a_4)$$

$$b_{d1} = \frac{1}{\sqrt{2}}(b_1 - b_2)$$

$$b_{c1} = \frac{1}{\sqrt{2}}(b_1 + b_2)$$

$$b_{d2} = \frac{1}{\sqrt{2}}(b_3 - b_4)$$

$$b_{c2} = \frac{1}{\sqrt{2}}(b_3 + b_4)$$

This linear combination of single-ended wave functions makes the transformation particularly transparent. One can define a simple transformation matrix to operate on single ended S-parameters to produce mixed-mode S-parameters. A few are shown here as an example (the entire set is in the appendix):

$$S_{d1d1} = \frac{1}{2}(S_{11} - S_{21} - S_{12} + S_{22})$$

$$S_{d1d2} = \frac{1}{2}(S_{13} - S_{23} - S_{14} + S_{24})$$

$$S_{d2d1} = \frac{1}{2}(S_{31} - S_{41} - S_{32} + S_{42})$$

$$S_{d2d2} = \frac{1}{2}(S_{33} - S_{43} - S_{34} + S_{44})$$

Eq. (4)

The simple linear relationship between the parameters should be evident. Just to reinforce the interpretation of these parameters: $S_{d2d1}$ is the differential output from composite port 2 (the old ports 3 and 4) ratioed against a differential drive into composite port 1 (the old ports 1 and 2). Similarly $S_{c2c2}$ would be the common-mode reflection off of composite port 2 ratioed against the common-mode signal applied to composite port 2.

The three port version of these is a straightforward simplification. Port 1 will remain single ended but ports 2 and 3 will become a balanced port.

$$\begin{bmatrix} b_1 \\ b_d \\ b_c \end{bmatrix} = \begin{bmatrix} S_{11} & S_{1d} & S_{1c} \\ S_{d1} & S_{dd} & S_{dc} \\ S_{c1} & S_{cd} & S_{cc} \end{bmatrix} \begin{bmatrix} a_1 \\ a_d \\ a_c \end{bmatrix}$$

Eq. (5)

$$a_d = \frac{1}{\sqrt{2}}(a_2 - a_3)$$

$$a_c = \frac{1}{\sqrt{2}}(a_1 + a_3)$$

$$b_d = \frac{1}{\sqrt{2}}(b_3 - b_4)$$

$$b_c = \frac{1}{\sqrt{2}}(b_2 + b_3)$$

Eq. (6)

$$S_{1d} = \frac{1}{\sqrt{2}}(S_{12} - S_{13})$$

$$S_{1c} = \frac{1}{\sqrt{2}}(S_{12} + S_{13})$$

$$S_{d1} = \frac{1}{\sqrt{2}}(S_{21} - S_{31})$$

$$S_{c1} = \frac{1}{\sqrt{2}}(S_{21} + S_{31})$$

Eq. (7)

To begin to look at these parameters, consider a simple balun: a three port device with one single ended port and (presumably) one differential port. This particular DUT is coaxial so an SOLT cal was used for both the base two port cal and for the three port extension (isolation and the optional thru line were skipped). Five of the single ended S-parameters ($S_{22}$ and $S_{33}$ are overlayed) are shown in Figure 16 and some of the corresponding mixed-mode parameters are shown in Figure 17. The single ended $S_{21}$ and $S_{31}$ show the expected 180 phase difference for nearly differential output behavior. The ports are also well-matched to 50 ohms. This leads to the important concept of differential and common mode port impedances. If both ports are at Z ohms in a single-ended sense, then the differential port impedance is 2Z while the common-mode port impedance is Z/2 based on the driving phases. Instrument settings are based on the single ended-impedances but this algorithm quickly determines the equivalent mixed mode port impedances.

The mixed mode parameters show the relatively high $S_{D1}$-differential transmission compared to the smaller $S_{C1}$-common-mode transmission. This difference of nearly 40 dB can be considered a measure of balance although other quantities are also used (e.g., $S_{31}/S_{21}$). Since $S_{C1}$ is proportional to $S_{21}+S_{31}$, the small resulting quantity should be expected. Note that this is an area of importance for cal stability: if the relationship between $S_{21}$ and $S_{31}$ drifts slightly over time, the resulting $S_{C1}$ can become considerably larger since it is dependent on the vector sum. Similarly since $S_{D1}$ is proportional to $S_{21}$-$S_{31}$, it should be large in magnitude due to the 180 degree phase difference. The balanced port is also well-matched to the respective differential and common-mode impedances for those drive types.

Another measure of balance mentioned above, $S_{31}/S_{21}$, is plotted in Figure 18. While this division can be performed using trace math, the method of inter-channel math is shown here: $S_{21}$ and $S_{31}$ are assigned to channels 1 and 2 respectively and channel 3 is defined to have the ratio of channel 2 to channel 1. This allows real-time tuning and permits trace memory to be used for other variables if need be. As can be seen the ratio is very close to magnitude 1 with phase of 180 degrees except at the higher frequencies where balance tends to degrade slightly. $S_{C1}$ is also in the plot for comparison and it also shows a rise at higher frequencies as would be expected.

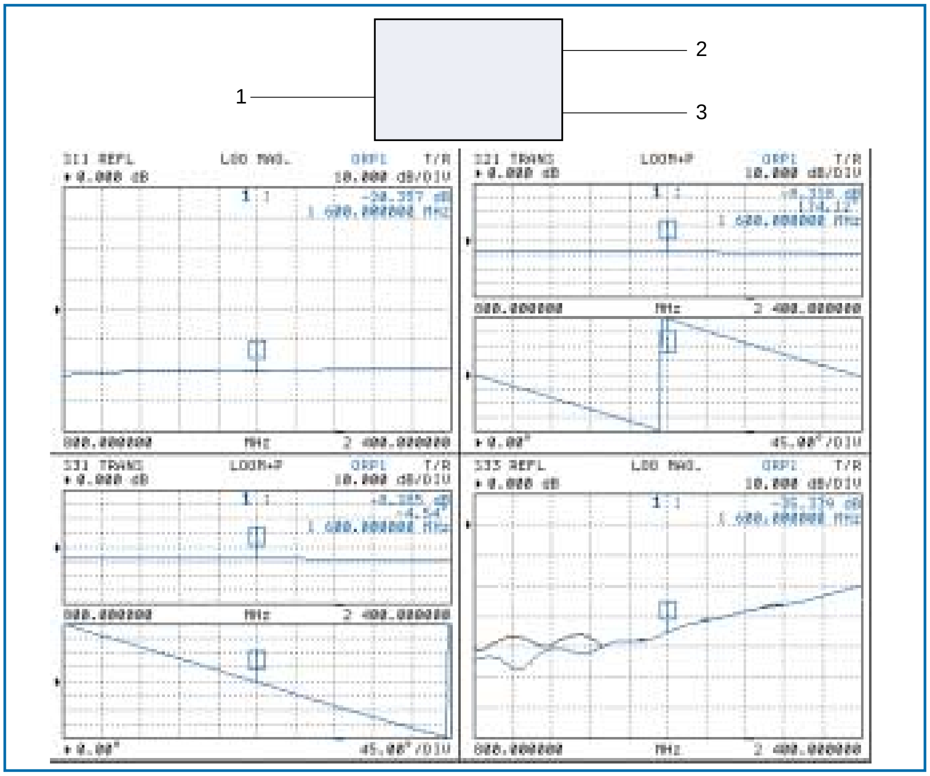


*Figure 16.* *Some single ended S-parameters of an example balun are shown here(in the lower right, $S_{33}$ is the darker trace, $S_{22}$ is the lighter trace). The ports are well-matched to 50 ohms and $S_{21}$ and $S_{31}$ are roughly equal in magnitude and 180 degrees out of phase as would be expected.*

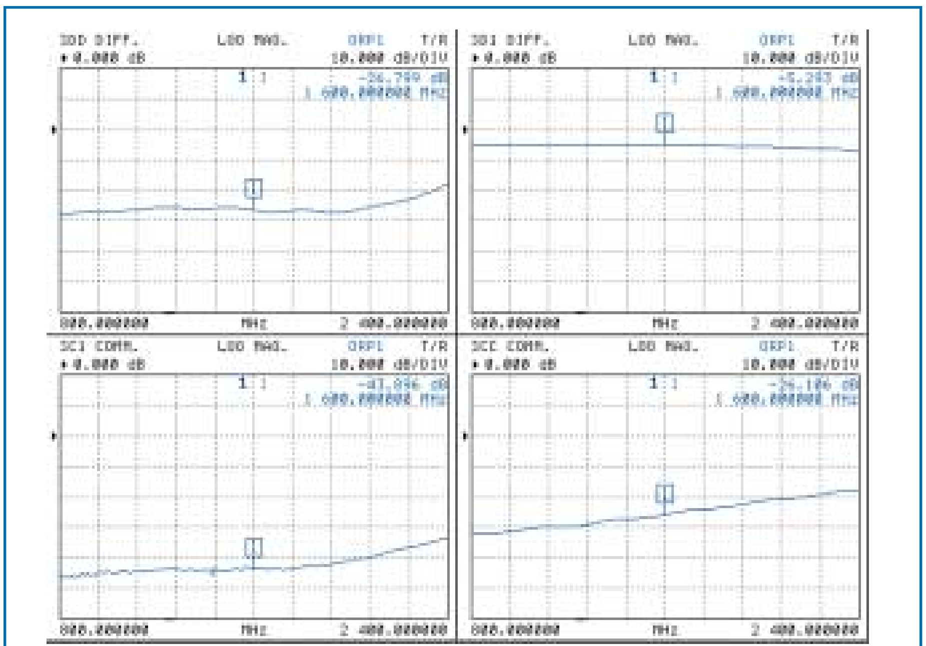

*Figure 17.* *Some mixed-mode S-parameters of the example balun are shown here. As might be expected, the differential transfer $S_{D1}$ is very large compared to the common-mode transfer $S_{C1}$.*

Next consider the example of a balanced amplifier. This four port active structure will be driven with -15 dBm at each single ended port to ensure linearity. Since the device was coaxial, an SOLT four port cal was performed from 10 MHz to 6 GHz (isolation and the redundant thru lines were skipped). The balanced structure of this particular amplifier starts to fall apart above about 2 GHz (transformers) so one might expect some anomalies at the higher frequencies. Some of the mixed mode parameters are shown in Figure 19. As one can see, the differential gain is between 13 and 17 dB at low frequencies with very good differential input match. The common mode gain is below about -40 dB at the low frequencies but rises at higher frequencies as the balance of the input transformer degrades. Similarly, mode conversion ($S_{C2D1}$ in this case) starts off below –20 dB below 2 GHz but increases at high frequencies due to decreasing balance. The user is reminded that if the DUT is being driven highly non-linearly, some assumptions inherent in the mixed-mode transformation break down and the results must be viewed with caution.

Consider the common-mode transfer a bit more carefully. Recall that this number represents the vector subtraction of a number of single ended S-parameters. Thus if even the phase of some of those parameters were to change (say from someone changing a setup), the magnitude of $S_{C2C1}$ could shift. To illustrate this, an extra 2.2 cm of transmission line (air dielectric) was added to port 1 only. At low frequencies, where the added phase length is small, there should be little change. At higher frequencies, the phase difference will result in poorer cancellation and a higher $S_{C2C1}$. This is shown in Figure 20 where the frequency range was restricted to the region of good balance so this effect could be separated out.

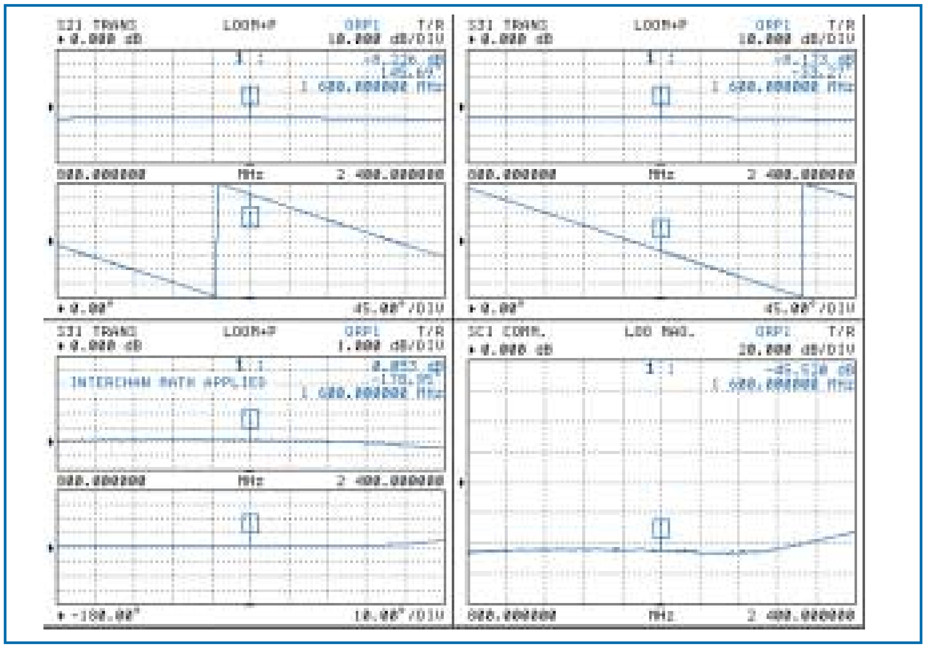

***Figure 18.*** *A measure of balance, $S_{31}/S_{21}$, is shown in the lower left hand corner (channel 3) of this plot. It is formed in this case by dividing channel 2 (upper right) by channel 1 (upper left). Ideal balance would be defined by a ratio of –1 (magnitude 1, angle 180 deg.) which is closely achieved until the higher frequencies. $S_{C1}$, plotted in the lower right, also shows this slight degradation.*

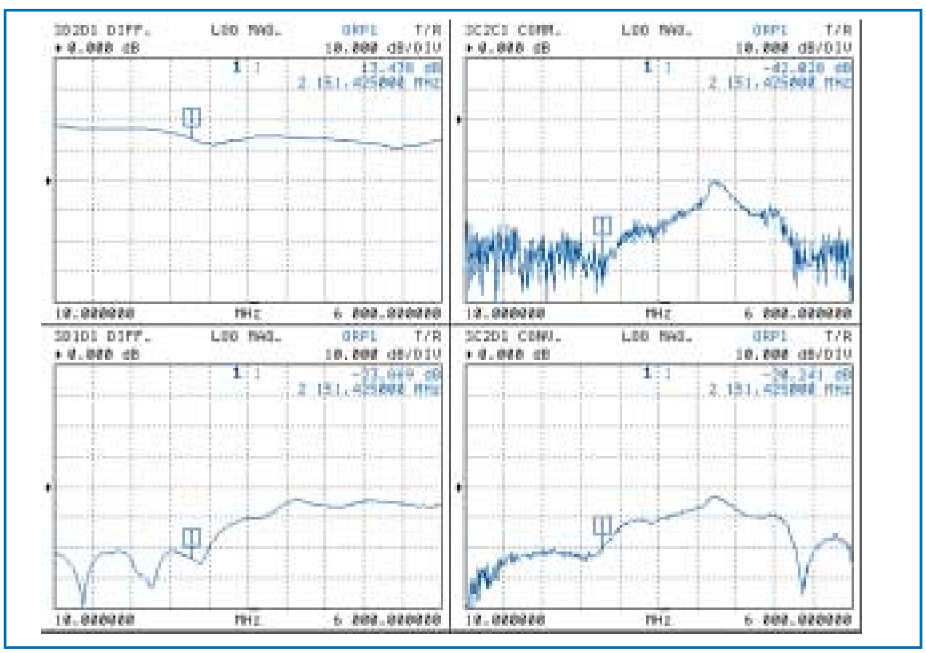

***Figure 19.*** *Some mixed-mode parameters for a balanced amplifier are shown here. Good balance is observed (common-mode transfer and mode-conversion low) below about 2 GHz; above that frequency, the performance of the device's input transformer starts to degrade.*

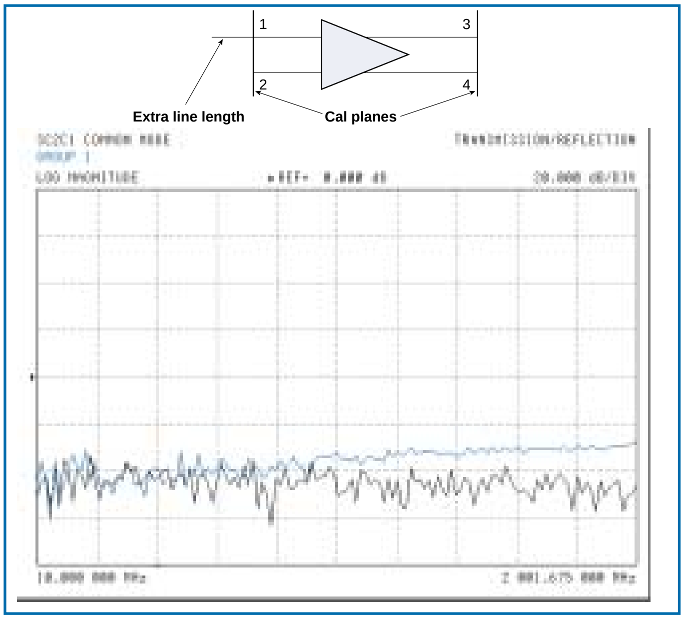


*Figure 20.* The effect of adding a line length to one port on measured balance (as shown through common-mode transfer) is illustrated here. As the frequency increases, the added line imputes a phase difference to some single-ended parameters over others resulting in an error on the mixed-mode parameter.

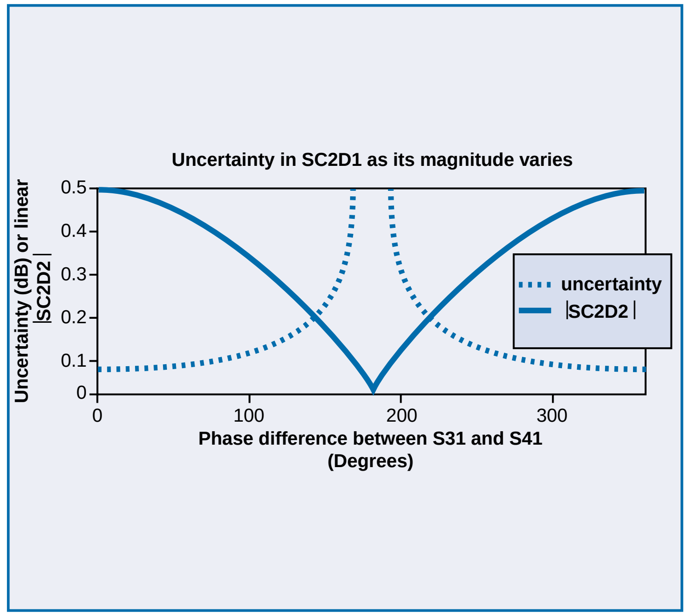


*Figure 21.* The uncertainty (in dB) in a mixed –mode parameter is a strong function of the magnitude of that parameter because of how they are computed.

## Uncertainty

The uncertainties in the four port measurement itself are straightforward extensions of those derived from the two and three port models. Associated with each port are residuals due to source and load match, directivity, and reflection tracking. Between every pair of ports is a residual due to transmission tracking. Other terms due to isolation and repeatability are often ignored in the analysis. The only real difference from a two port model is that there are multiple load match terms and each is dependent on path isolation.

As an example, consider the measurement of $S_{11}$. The primary factors affecting this measurement (ignoring noise floor and compression effects) will be residual directivity errors ($\Delta ed1$), source match errors ($\Delta ep1s$), reflection tracking ($\Delta et11$) and load match ($\Delta epnL$). This simplified analysis also ignores multiple reflections between a series of ports since they will be at most 2nd order effects.

$$err \approx \Delta ed1 + S_{11} \cdot \Delta et11 + S_{11}^{2} \cdot \Delta ep1S + S_{21}S_{12} \cdot \Delta ep2L + S_{31}S_{13} \cdot \Delta ep3L + S_{41}S_{14} \cdot \Delta ep4L \qquad \text{Eq. (8)}$$

Clearly if some of the transmission terms are small, the load match quality becomes less important as observed before. Since the mixed mode terms are simple linear combinations of the normal S-parameters, the uncertainty in a mixed-mode parameter is easy to compute:

$$S_{c2d1} = \frac{1}{2}(S_{31} + S_{41} - S_{32} - S_{42}) \qquad \text{Eq. (9)}$$

$$\frac{\Delta S_{c2d1}}{S_{c2d1}} \le \frac{|\Delta S_{31}| + |\Delta S_{41}| + |\Delta S_{32}| + |\Delta S_{42}|}{S_{31} + S_{41} - S_{32} - S_{42}} \qquad \text{Eq. (10)}$$

Where linear absolute magnitudes can be used for a worst case computation. The caveat here is that this is in linear terms while uncertainties are often more useful in a log form. This has a surprising effect for some parameters. As an example, consider the case where the single-ended uncertainties are fixed at .03 dB, $S_{32}$ and $S_{42}$ are zero and $S_{31}$ and $S_{41}$ are allowed to vary. For the purposes of an example, the magnitude of $S_{31}$ and $S_{41}$ will be fixed at 0.5 and the phase will be allowed to vary. As one can see in Figure 21, when the phase of the single ended parameters is such that $|S_{C2D1}|$ is small, the uncertainty (in dB terms) explodes. This follows from Eq. 10 since the numerator is largely fixed but the denominator can get small.

Since the desired quantity ($S_{c2d1}$ in this case) is very small, the relative uncertainty can be quite high. This is somewhat intuitive since the computation is subtracting nearly equal numbers. The user is therefore cautioned that cal stability will be extremely critical on the smaller mixed mode parameters and the uncertainties will be relatively high. The biggest source of problems will often be cable changes with flexure or connector repeatability.

## Conclusions

Several introductory aspects of multiport S-parameters have been presented. The calibration methods provided are fairly natural extensions of the two port calibration methods that the user may be familiar with. In many multiport environments, the presentation of mixed-mode S-parameters (common-mode and differential drive/output) is important. The definitions of these parameters have been discussed along with some information on interpretation and uncertainty analysis.

# Appendix

### Cal term computations

While it is beyond the scope of this document to describe calibration term computation for all of the algorithms (see [3]-[6] for more), it will be useful to introduce some of the concepts and notation. Three of the terms types are local to a port and are easily found using one port measurements:

*edn*: directivity of the n[th] port. If a perfect load is connected to the port, the measurement of $S_{nn}$ yields this term. (=dn in Figure 9)

*epnS*: Source match of the n[th] port. (=mn in Figure 9 when port n is driving)

*etnn*: Reflection tracking of the nth port. (=tan*tbn in Figure 9)

There are many ways to find these two term types as discussed in the references. In SOLT for example, the short and open measurements (whose standards have known reflection coefficients, $\Gamma_{short}$ and $\Gamma_{open}$, from factory characterization) yield two equations in two unknowns (*etnn* and *epnS*):

$$S_{nn}^{short,measured} = edn + \frac{etnn \cdot \Gamma_{short}}{1 - epnS \cdot \Gamma_{short}}$$
$$S_{nn}^{short,measured} = edn + \frac{etnn \cdot \Gamma_{open}}{1 - epnS \cdot \Gamma_{open}} \qquad \text{Eq. (11)}$$

The two remaining term types generally require some type of port interconnect

*etmn*: Transmission tracking from port n to port m (=tan*tbm in Figure 9)

*epnL*: Load match at port n. (=mn in Figure 9 when port n is not driving)

Note that load match is defined as the residual mismatch at a port when it is NOT driving (as opposed to source match). This distinction is ignored in some cal algorithms and generally that causes difficulties with practical VNAs. Generally a thru or some line length will be connected between a pair of ports to start generating these terms. While there are many permutations, consider the connection of a line of electrical length θ connected between ports 1 and 2. All four S-parameters can be measured:

$$S_{11}^{measured,\,thru} = ed1 + \frac{et11 \cdot ep2L \cdot e^{-j2\theta}}{1 - ep1S \cdot ep2L \cdot e^{-j2\theta}}$$
$$S_{21}^{measured,\,thru} = \frac{et21 \cdot e^{-j2\theta}}{1 - ep1S \cdot ep2L \cdot e^{-j2\theta}}$$
$$S_{12}^{measured,\,thru} = \frac{et12 \cdot e^{-j2\theta}}{1 - ep1L \cdot ep2S \cdot e^{-j2\theta}} \qquad \text{Eq. (12)}$$
$$S_{22}^{measured,\,thru} = ed2 + \frac{et22 \cdot ep1L \,.\, e^{-j2\theta}}{1 - ep1L \cdot ep2S \cdot e^{-j2\theta}}$$

If the first three term types have already been determined, then there are four unknowns (*ep1L*, *ep2L*, *et21*, and *et12*) in the above four equations so they can all be solved for. Isolation is normally performed as a separate step in which all ports are terminated and the isolation term is the residual $S_{mn}$. Since algorithms like LRL do not determine the first three term types in the same way, one might think this presents a problem. The above equation also hints that thru lines must be connected between every port pair. This brings up the subject of redundancy that was discussed earlier.

The key lies in the tracking terms and how they are defined (see Figure 5). Since the reflection and transmission tracking terms overlap, they are not entirely independent. The following is the relationship that one can pull directly off of Figure 5:

$$etkn \cdot etnl = etnn \cdot etkl \qquad \text{Eq. (13)}$$

To see how this is used, consider the redundant thru line problem. We have connected a thru between 1-2, 1-3 and 1-4 in the most reduced case. Presumably the first three term types have been computed for all four ports. The thru connects allow us to compute all of the load match terms using Eq. 12 and all transmission tracking terms except for *et32, et23, et24, et42, et34* and *et43*. These last tracking terms can be found with the following forms of Eq. 13:

$$et43 = \frac{et41 \cdot et13}{et11} \; (k = 4, l = 3, n = 1)$$
$$et32 = \frac{et31 \cdot et12}{et11} \; (k = 3, l = 2, n = 1) \qquad \text{Eq. (14)}$$
$$\text{etc.}$$

A similar concept is used in the LRL family of calibrations to remove unknowns but the details are left to the references. The important idea is that the interdependency of the tracking terms allows for fewer measurements than may appear obvious.

Mixed mode equations

For completeness, the equations for all of the mixed mode terms (both three and four port variants) are presented below.

The differential-to-differential terms:

$$S_{d1d1} = \frac{1}{2}(S_{11} - S_{21} - S_{12} + S_{22})$$
$$S_{d1d2} = \frac{1}{2}(S_{13} - S_{23} - S_{14} + S_{24})$$
$$S_{d2d1} = \frac{1}{2}(S_{31} - S_{41} - S_{32} + S_{42}) \qquad \text{Eq. (15)}$$
$$S_{d2d2} = \frac{1}{2}(S_{33} - S_{43} - S_{34} + S_{44})$$

The common mode-to-common mode terms:

$$S_{c1c1} = \frac{1}{2}(S_{11} + S_{21} + S_{12} + S_{22})$$

$$S_{c1c2} = \frac{1}{2}(S_{13} + S_{23} + S_{14} + S_{24})$$

$$S_{c2c1} = \frac{1}{2}(S_{31} + S_{41} + S_{32} + S_{42})$$

$$S_{c2c2} = \frac{1}{2}(S_{33} + S_{43} + S_{34} + S_{44})$$

Eq. (16)

The common mode-to-differential terms:

$$S_{d1c1} = \frac{1}{2}(S_{11} - S_{21} + S_{12} - S_{22})$$

$$S_{d1c2} = \frac{1}{2}(S_{13} - S_{23} + S_{14} - S_{24})$$

$$S_{d2c1} = \frac{1}{2}(S_{31} - S_{41} + S_{32} - S_{42})$$

$$S_{d2c2} = \frac{1}{2}(S_{33} - S_{43} + S_{34} - S_{44})$$

Eq. (17)

and the differential-to-common mode terms:

$$S_{c1d1} = \frac{1}{2}(S_{11} + S_{21} - S_{12} - S_{22})$$

$$S_{c1d2} = \frac{1}{2}(S_{13} + S_{23} - S_{14} - S_{24})$$

$$S_{c2d1} = \frac{1}{2}(S_{31} + S_{41} - S_{32} - S_{42})$$

$$S_{c2d2} = \frac{1}{2}(S_{33} + S_{43} - S_{34} - S_{44})$$

Eq. (18)

For a three port DUT (where port 1 is defined to be the single-ended port), the equations are:

$$S_{1d} = \frac{1}{\sqrt{2}}(S_{12} - S_{13})$$

$$S_{1c} = \frac{1}{\sqrt{2}}(S_{12} - S_{13})$$

$$S_{d1} = \frac{1}{\sqrt{2}}(S_{21} - S_{31})$$

$$S_{c1} = \frac{1}{\sqrt{2}}(S_{21} - S_{31})$$

$$S_{dd} = \frac{1}{2}(S_{22} - S_{23} - S_{32} + S_{33})$$

$$S_{cc} = \frac{1}{2}(S_{22} + S_{23} + S_{32} + S_{33})$$

$$S_{dc} = \frac{1}{2}(S_{22} + S_{23} - S_{32} - S_{33})$$

$$S_{cd} = \frac{1}{2}(S_{22} - S_{23} + S_{32} - S_{33})$$

Eq. (19)

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