Noise in Antennas

Thus far we have examined how to calculate the power radiated from an antenna and, using the Friis transmission formula, how to calculate the received power at the other end of a communication link. However, the received signal power is meaningless unless compared with the power received from unwanted sources over the same bandwidth. Such noise sources include thermal radiation from the earth and sky, cosmic background radiation, and random thermal processes in the receiving system. In today's wireless environment, additional noise due to nonstationary radio frequency interference from pagers, cellular phones, *etc.*, often needs to be considered, but in this analysis we will concentrate on natural sources only.

1 Natural Sources Characterized

1.1 Brightness

Consider the source/receiver configuration shown in Fig. 1. The *brightness* of the source is

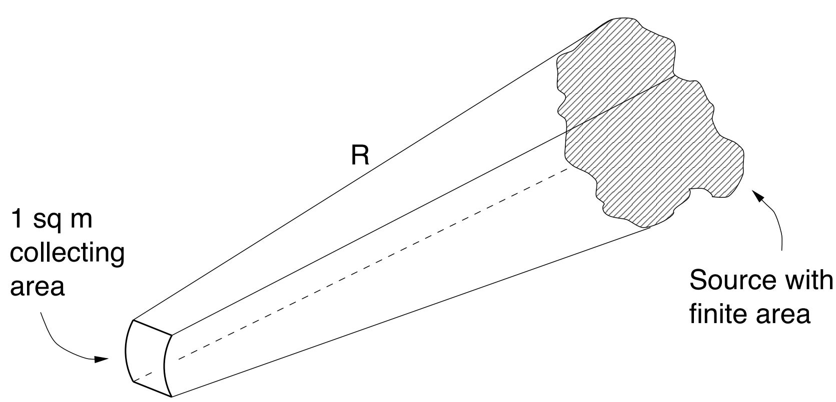

Figure 1: Radiation from a natural source.

defined to be the electromagnetic flux density (power per unit area) at the receiver per unit solid angle of source. By dimensional analysis,

$$\left(\frac{\text{Power}}{\text{m}^2}\right)_{\text{at collector}} \left(\frac{1}{\text{m}^2/\text{R}^2}\right)_{\text{at source}} = \frac{\text{Power} \cdot \text{R}^2}{\text{m}^2 \cdot \text{m}^2} = \text{Brightness.}$$

From the dimensional analysis we can see that an equivalent definition of brightness is power per unit area of source per unit solid angle of receiver. *Monochromatic brightness* is brightness per unit frequency. Often the term "brightness" only is used, without the monochromatic qualifier; the meaning is usually clear from the context.

Contrast the definition of brightness with that of flux density: power flow per unit area. Brightness is suitable for extended sources, while flux is suitable for point sources, as shown in Fig. 2. Each cross-sectional area (S, S', S'') intercepts the same total power, but the flux

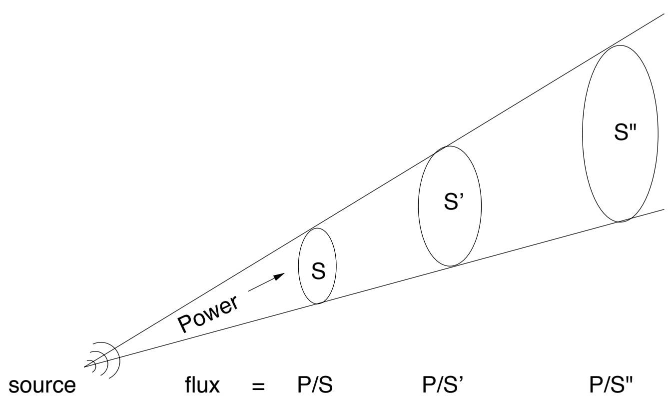

Figure 2: Power flux from a point source.

density decreases as energy propagates away from the source. The power intercepted by a receiver at any point is the flux density times the effective aperture of the receiver, a relation which we have already used many times when calculating the power received by an antenna.

1.2 Blackbody Radiation

A *blackbody* is an idealized body which absorbs all electromagnetic energy at all wavelengths impinging upon it. Since an object in thermal equilibrium emits energy at the same rate that it absorbs energy, a blackbody also radiates electromagnetic energy at all wavelengths.

Although no blackbodies exist in nature, many objects (*e.g.*, the sun) behave approximately like blackbodies over a range of frequencies. From Planck's Law, the total power per unit frequency radiated isotropically from a unit area of blackbody at temperature T is

$$P_\nu = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}, \tag{1}$$

where ν is the frequency, h is Planck's constant (6.63×10^{-34} Joule sec), and k is Boltzmann's constant (1.38×10^{-23} Joule Kelvin^{-1}). The MKS unit used for P_ν is the Jansky, named after Karl Jansky, who discovered early in the 1930's that the galaxy is a source of radio emissions and hence became the "father of radio astronomy" (1 Jy $= 10^{-26}$ W m^{-2} Hz^{-1}). At long wavelengths, where $h\nu \ll kT$ (or, equivalently, $\nu \ll 2\times10^{10}\,T$), the Rayleigh-Jeans approximation gives

$$P_\nu \approx \frac{2\pi h\nu^3}{c^2}\frac{kT}{h\nu} = \frac{2\pi kT}{\lambda^2}. \tag{2}$$

What is the (monochromatic) brightness of a blackbody? Consider a spherical blackbody of radius a. Conservation of energy requires that the power emitted from the blackbody is equal to the power intercepted by a concentric sphere of radius R. Since the power is radiated from the blackbody isotropically, we can write

$$4\pi a^2 P_\nu = 4\pi R^2 P'_\nu,$$

where P_ν is the power (per unit area per unit frequency) leaving the surface of the blackbody and P'_ν is the power at any point at radius R. Rearranging, we get

$$\frac{P'_\nu}{P_\nu} = \left(\frac{a}{R}\right)^2,$$

and substituting for P_ν from Eq. 1,

$$P'_\nu = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\left(\frac{a}{R}\right)^2 \text{ Watts m}^{-2}\text{ Hz}^{-1}. \tag{3}$$

Equation 3 gives the received flux density per unit frequency at radius R. Then, by definition, the brightness is the received flux density per unit frequency per unit solid angle of source:

$$\begin{aligned} B_\nu &= \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\left(\frac{a}{R}\right)^2 \frac{1}{\pi a^2/R^2} \\ &= \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\left(\frac{a}{R}\right)^2 \frac{R^2}{\pi a^2} \\ &= \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \text{ Watts m}^{-2}\text{ Hz}^{-1}\text{ sr}^{-1}, \end{aligned} \tag{4}$$

which differs from Eq. 1 dimensionally and by a factor of π in magnitude. The corresponding Rayleigh-Jeans approximation is

$$B_\nu \to \frac{2kT}{\lambda^2}, \qquad \frac{h\nu}{kT} \ll 1. \tag{5}$$

1.3 Antenna Temperature

How do we calculate the power captured from an extended source? In Fig. 3 we show a narrow beam antenna whose pattern is confined to a solid angle $d\Omega$. The beam intercepts

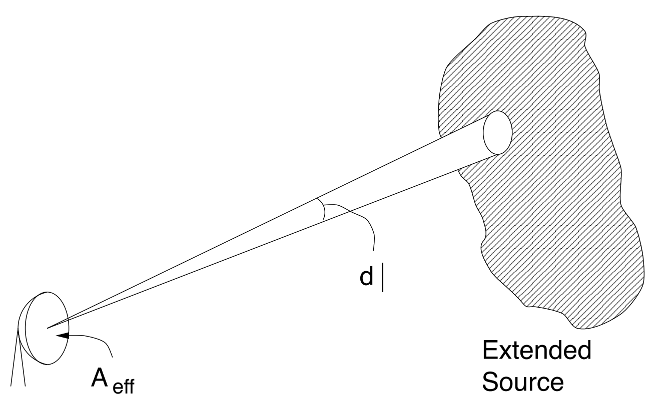

Figure 3: Power reception from an extended source.

an extended brightness source, which could be either (i) noise background, or (ii) the signal of interest! If the antenna is pointed toward the (θ, ϕ) direction, how much power is received in bandwidth ΔB?

$$\begin{aligned} P_{\text{rec}}(\theta, \phi) &= \frac{1}{2} B_\nu(\theta, \phi) d\Omega A_{\text{eff}}(\theta, \phi) \Delta B \\ &= \frac{1}{2} \frac{2kT(\theta, \phi)}{\lambda^2} d\Omega A_{\text{eff}}(\theta, \phi) \Delta B \\ &= \frac{kT(\theta, \phi) G(\theta, \phi) d\Omega \Delta B}{4\pi}, \end{aligned}$$

where

$$G(\theta, \phi) = \frac{4\pi A_{\text{eff}}(\theta, \phi)}{\lambda^2}$$

is the gain of the antenna. Note that an extra factor of 1/2 appears in the equations above because antennas can only extract power in a single polarization. Total random power is partitioned between two (any two) orthogonal polarizations.

The total power received by the antenna is

$$\begin{aligned} P_{\text{total received}} &= \frac{k\Delta B}{4\pi} \int_{4\pi} G(\theta, \phi) T(\theta, \phi) d\Omega \\ &= k \Delta B T_{\text{ant}} \end{aligned} \tag{6}$$

where

$$T_{\text{ant}} = \frac{1}{4\pi} \int_{4\pi} G(\theta, \phi) T(\theta, \phi) d\Omega. \tag{7}$$

Significant contributions to the effective antenna temperature, T_{ant}, can come from a "large" product GT over small "areas of sky" or a "small" GT over large "areas of sky."

2 Noise in Circuits

2.1 Nyquist's Law

Referring to the left portion of Fig. 4, what is the noise voltage associated with a resistance R at temperature T? From *Nyquist's Law*, the mean-square voltage due to random thermal

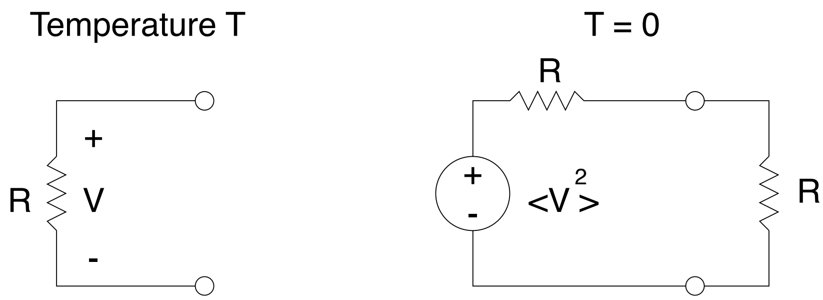

Figure 4: Thermal noise in resistors: Circuit models.

processes is

$$\langle V^2 \rangle = 4kTRB \tag{8}$$

where B is the bandwidth. From the equivalent circuit at the right of Fig. 4, the power transfer to a matched load is

$$\frac{\langle V^2 \rangle}{4R} = kTB, \tag{9}$$

which is *independent* of R. Table 1 gives a few example values for $\langle V^2 \rangle^{1/2}$ and kTB for various values of R, T, B.

2.2 Noise in Amplifiers

Now consider the noise in an amplifier, as suggested by the black box of Fig. 5. The noise power fed into the amplifier is $N_{\rm in} = kT_s B$, and we denote the noise power coming out of the amplifier as $N_{\rm out}$. (Note that if $R_s = R_{\rm rad}, T_s = T_{\rm ant}$.) Since an amplifier at a temperature

T (K)	B (kHz)	kTB (dBm)	R (Ω)	$\langle V^2 \rangle^{1/2}$ (μV)
300	5	-197	5×10^5	6.4
"	"	"	50	0.064
30	"	-207	5×10^5	2.0
"	"	"	50	0.02

Table 1: Sample resistive noise calculations.

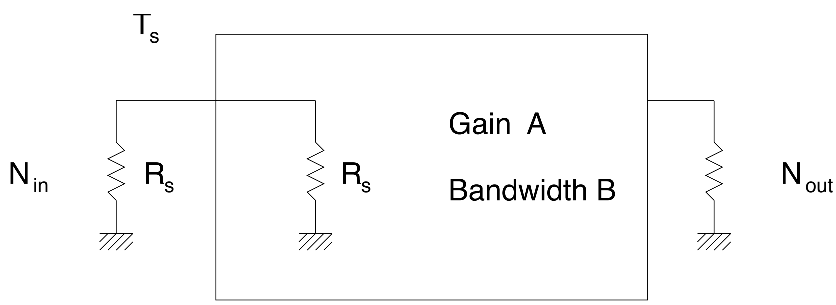

Figure 5: Noise in amplifiers.

greater than 0 Kelvin must supply some noise, $N_{\text{out}} > N_{\text{in}}$. We can write N_{out} as

$$N_{\text{out}} = (kT_{\text{eq}}B) \cdot A = k(T_s + T_{\text{amp}})B \cdot A \tag{10}$$

The amplifier temperature, T_{amp}, is treated as though the noise were added at the front end of the amplifier, as shown in Fig. 6. The *noise figure* of an amplifier is the signal-to-noise

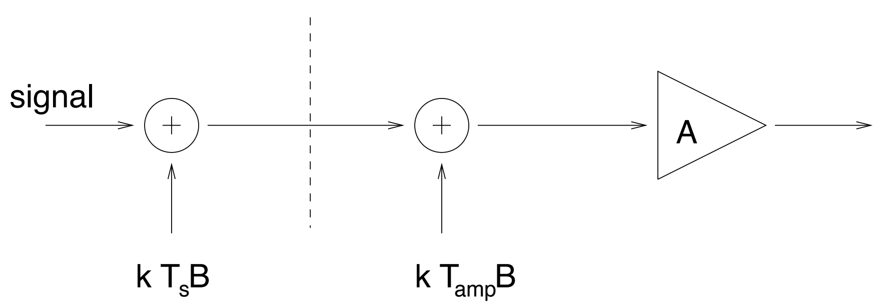

Figure 6: Model for amplifier noise.

ratio at the amplifier input divided by the signal-to-noise ratio at the output:

$$F = \frac{(S/N)_{\rm in}}{(S/N)_{\rm out}} = \frac{S/(kT_sB)}{S/(kT_{\rm eq}B)} = 1 + \frac{T_{\rm amp}}{T_s} > 1 \tag{11}$$

Note that the definition of noise figure depends on T_s, which is usually taken as 290 K, corresponding approximately to room temperature.

For low noise amplifiers (LNAs) it is much easier to work with temperatures alone, rather than noise figures. For example, for the amplifier chain shown in Fig. 7, we can write the

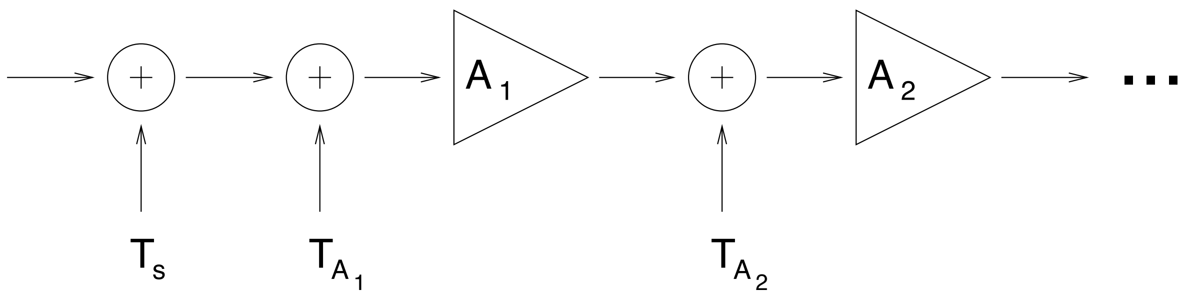

Figure 7: Amplifier chain.

signal-to-noise ratio as

$$\begin{aligned} \mathrm{SNR} &= \frac{S \cdot A_1 \cdot A_2 \cdots}{kB((((T_s + T_{A1})A_1 + T_{A2})A_2 + \cdots) \cdots)} \\ &= \frac{S}{T_s + T_{A1} + T_{A2}/A_1 + \cdots}. \end{aligned} \tag{12}$$

Thus, the noise contribution of each amplifier is effectively reduced by the gain of the leading (preceding) amplifiers.

2.3 Lossy elements

How do we account for the effects of “lossy” elements, such as attenuators, transmission lines, *etc.*? In Fig. 8m L represents the fraction of power *lost* in attenuation. The signal

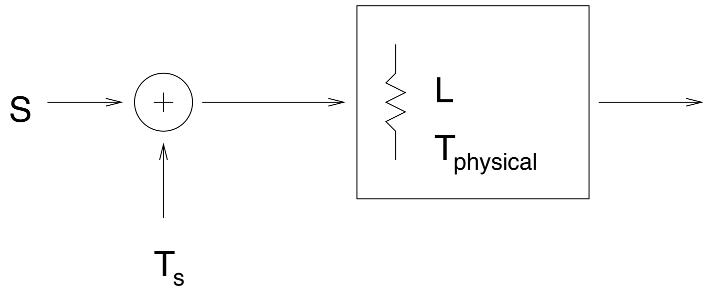

Figure 8: Noise in lossy elements.

power after attenuation is then $S(1-L)$ and the noise power is

$$N_{\text{out}} = \underbrace{kT_s(1-L)B}_{\text{noise passed through load/attenuator ``L''}} + \underbrace{kT_{\text{physical}}LB}_{\text{noise added by ``L''}} .$$

The attenuator noise figure is

$$\begin{aligned} F = \frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} &= \frac{S/(kT_sB)}{S(1-L)/(k(T_s(1-L)+T_{\text{physical}}L)B)} \\ &= \frac{T_s(1-L)+T_{\text{physical}}L}{T_s(1-L)} \\ &= 1+\frac{T_{\text{physical}}L}{T_s(1-L)}. \end{aligned} \tag{13}$$

If we let α denote the fraction of power *transmitted* through the attenuator (*i.e.*, $\alpha = 1-L$), then we can write another commonly used expression for the attenuator noise figure:

$$\begin{aligned} F|_{T_{\text{physical}}=T_s} &= 1+\frac{1-\alpha}{\alpha} \\ &= \frac{\alpha+1-\alpha}{\alpha} \\ &= \alpha^{-1}. \end{aligned} \tag{14}$$

For example, a 10 dB attenuator ($\alpha = 0.1$) has a noise figure of 10 dB.

We can rewrite the output SNR as

$$
\begin{aligned}
(S/N)_{\text{out}} &= \frac{S(1-L)}{k(T_s(1-L)+T_{\text{physical}}B} \\
&= \frac{S}{k\left(T_s+T_{\text{physical}}\left(\frac{1}{1-L}\right)\right)B}.
\end{aligned} \tag{15}
$$

Hence the equivalent front-end temperature of the attenuator is

$$
T_{\text{attenuator}} = T_{\text{physical}}\frac{L}{1-L} \longrightarrow \infty \qquad \text{as } L \to 1. \tag{16}
$$

We are now in a position to calculate the signal-to-noise ratio for an antenna connected to an amplifier:

$$
SNR = \frac{P_{\text{r}}}{kT_{\text{sys}}B} = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2}\frac{1}{kT_{\text{sys}}B}, \tag{17}
$$

where

$$
T_{\text{sys}} = \underbrace{T_{\text{ant}}}_{\frac{1}{4\pi}\int_{4\pi} G_r T_{\text{sky}}\, d\Omega} + \underbrace{T_{\text{feed}}}_{T_{\text{physical}}\frac{L}{1-L}} + \underbrace{T_{A1} + \frac{T_{A1}}{A_1} + \frac{T_{A2}}{A_1 A_2} + \cdots}_{\text{amplifier dependent}}.
$$

Due to cosmic background radiation, $T_{\text{ant}} \geq 3$ K. For fairly efficient systems, values for T_{sys} might be 20 K (maser), 50 K (FEMT), 100 K (FET). However, other sources of loss not directly associated with the antenna/receiver system might also need to be considered. For example, clouds, raindrops, atmospheric gases, *etc.*, all can act as attenuators for satellite signals. Some of the signal power goes into heating the clouds, and some of the heat from the clouds is radiated into the receiving antenna. This loss can be incorporated into our model as another “L”, as before.

Suppose you are running a high performance, high efficiency ground station with $T_{\text{sys}} \approx$

30 K, operating at X-band. A stratus cloud might give 1 dB loss (*i.e.*, $L = 0.2$). Then

$$P_{\text{sig}} \longrightarrow 0.8 P_{\text{sig}}$$

and

$$T_{\text{sys}} \longrightarrow 0.8(30\ \text{K}) + 0.2(300\ \text{K}) = 84\ \text{K}.$$

The ratio of SNRs with and without the cloud is

$$\frac{(S/N)_{\text{cloud}}}{(S/N)_{\text{no cloud}}} = \frac{0.8/84}{1/30} \approx -6\ \text{dB}.$$

Even though you've only lost 1 dB of signal, your SNR decreases by 6 dB!!!

On the other hand, what if T_{ant} is very large, so that

$$T_{\text{ant}} \gg T_{\text{physical}} \frac{L}{1-L} + \sum_n T_{An}?$$

Then $T_{\text{sys}} \approx T_{\text{ant}}$ is dominated by noises propagated from the outside, rather than that introduced by your receiver. This is the case for AM radio ($f_c \approx 1$ MHz); as long as $P_r/(kT_{\text{ant}}B) \gg 1$, you can use almost any receiving system. This is why a very short antenna ($L/\lambda \approx 1/300$) works well enough, although R_{rad} for a short antenna is very low and very difficult to match to the receiver!

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